Fair Pay Rise Problem

# The Problem

Company X[[1]](#footnote-1) has several employees each on a different salary over a significant range (say £16k to £250k). The company has had a successful year and wants to give all employees a “fair” pay rise to the effect of percent overall. The question is how to make it fair?

## Example:

Below is Company X’s employee list with current salaries and the total sum of those salaries:

|  |  |
| --- | --- |
| **Name[[2]](#footnote-2)** | **Current Salary** |
| Andrew | £ 16,300 |
| Beatrice | £ 20,000 |
| Cason | £ 30,000 |
| Dalton | £ 30,000 |
| Emma | £ 30,000 |
| Frank | £ 40,000 |
| Gilbert | £ 40,000 |
| Hayleigh | £ 60,000 |
| Ian | £ 150,000 |
| Jenna | £ 250,000 |
| **Total** | **£ 666,300** |

Table 1

The company is willing to give the equivalent of 10% raise to everyone in the company. This means it’s happy to increase the Total Salary Budget by 10%:

Equation 1

Normally most companies would give each employee the same 10% raise which would results in:

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Current Salary** | **Employee**  **Raise** | **New Salary** |
| Andrew | £16,300 | £1,630 | £17,930 |
| Beatrice | £20,000 | £2,000 | £22,000 |
| Cason | £30,000 | £3,000 | £33,000 |
| Dalton | £30,000 | £3,000 | £33,000 |
| Emma | £30,000 | £3,000 | £33,000 |
| Frank | £40,000 | £4,000 | £44,000 |
| Gilbert | £40,000 | £4,000 | £44,000 |
| Hayleigh | £60,000 | £6,000 | £66,000 |
| Ian | £150,000 | £15,000 | £165,000 |
| Jenna | £250,000 | £25,000 | £275,000 |
| **Total** | **£666,300** | **£66,630** | **£732,930** |

Table 2

But the question being asked by Company X is

Is this fair?

Notice that Jenna’s pay rise of £25,000 is more than Andrew’s new salary of £17,930.

Not only that but Jenna’s pay rise is **5 times** more than most of the employees at the company.

Notice that under this scheme the ratio between employees pay stays static over time (i.e. Hayleigh will always earn 3 times more than Beatrice if each year they get the same percentage raise).

How else could the company decide to divide up the £66,630 pay rise between the employees?

# A few words on Fairness

What is “fair” in a moral sense is a matter for debate by philosophy graduates. But for a moment consider:

*The average wage of a population is strongly based on the highest earners. And the price of goods for a given population must be tied to the total income of that population. If all earners in the population get the same percentage pay-rise year on year then has the lowest paid worker gained anything overall? – No.*

In this paper “fair” is taken to mean that the salaries should aim to converge over time to a common salary for all employees while not negatively impact any one employee.

This is considered good for a few reasons:

1. The end goal is equal pay for all.
2. If you negotiate a bad salary to can be assured that over time that salary will come to be aligned with other employees.
3. Employers can offer a low starting salary to new employees but the longer they stay the more equal their pay becomes.
4. The only way for the biggest earners to get a big raise is to reduce the discrepancy in pay between all employees.

For the purposes of initial discussion “pay for performance” is not considered. It is assumed the company has one pot of money put aside for pay rises which it wishes to divide fairly among its employees.

# Proposed Solutions

## Option 1. Equal Pay Rise

A simple solution would be to take the Company Raise and divide it equally between all employees.

Equation 2

Each Employee gets £6,663 extra for their new Salary.

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Current Salary** | **Employee**  **Raise** | **New Salary** |
| Andrew | £16,300 | £6,663 | £22,963 |
| Beatrice | £20,000 | £6,663 | £26,663 |
| Cason | £30,000 | £6,663 | £36,663 |
| Dalton | £30,000 | £6,663 | £36,663 |
| Emma | £30,000 | £6,663 | £36,663 |
| Frank | £40,000 | £6,663 | £46,663 |
| Gilbert | £40,000 | £6,663 | £46,663 |
| Hayleigh | £60,000 | £6,663 | £66,663 |
| Ian | £150,000 | £6,663 | £156,663 |
| Jenna | £250,000 | £6,663 | £256,663 |
| **Total** | **£666,300** | **£66,630** | **£732,930** |

Table 3

This does seem fairer on the surface.

Both Jenna and Andrew get the same raise.

Jenna no longer gets 5 times more of a raise than most employees.

The ratio between employees pay will reduce over time. (i.e. Hayleigh earns 3 times more than Beatrice with their Current Salaries but only 2.5 times more under their New Salaries).

But the pay gap between employees will remain constant over time. (i.e. Frank will always earn £10,000 more than Emma).

It is interesting to view this Equal Pay Rise scheme as a percentage for each employee:

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Current Salary** | **Employee**  **Raise** | **Raise as %** |
| Andrew | £16,300 | £6,663 | 40.88% |
| Beatrice | £20,000 | £6,663 | 33.32% |
| Cason | £30,000 | £6,663 | 22.21% |
| Dalton | £30,000 | £6,663 | 22.21% |
| Emma | £30,000 | £6,663 | 22.21% |
| Frank | £40,000 | £6,663 | 16.66% |
| Gilbert | £40,000 | £6,663 | 16.66% |
| Hayleigh | £60,000 | £6,663 | 11.11% |
| Ian | £150,000 | £6,663 | 4.44% |
| Jenna | £250,000 | £6,663 | 2.67% |
| **Total** | **£666,300** | **£66,630** |  |

Table 4

Andrew gets a 40.88% pay rise under this scheme, a huge increase in salary that will mean a lot to him[[3]](#footnote-3).

On the down side Jenna only gets a 2.67% which doesn’t seem like a lot. But remember Jenna earns 15 times more than Andrew already, so will she really miss it?

All bar two employees (Ian and Jenna) get more than the 10% raise that would have been expected.

## Option 2. Raise of Others

We could view a “fair” pay rise scheme as one which is designed to move all salaries in the direction of equal.

A method to do this is to take the mean salary of the rest of the company and calculate a raise based on that mean. Over time this has the effect of brining all employees to the same salary.

Equation 3

We can extract from the above the Raise for Employee *n* as:

Equation

This can be described as the raise for the mean employee not including employee *n*. By removing employee *n* from the calculated Mean it means their salary will not be able to skew the Mean up or down.

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Current Salary** | **Employee**  **Raise** | **New Salary** |
| Andrew | £16,300 | £7,222 | £23,522 |
| Beatrice | £20,000 | £7,181 | £27,181 |
| Cason | £30,000 | £7,070 | £37,070 |
| Dalton | £30,000 | £7,070 | £37,070 |
| Emma | £30,000 | £7,070 | £37,070 |
| Frank | £40,000 | £6,959 | £46,959 |
| Gilbert | £40,000 | £6,959 | £46,959 |
| Hayleigh | £60,000 | £6,737 | £66,737 |
| Ian | £150,000 | £5,737 | £155,737 |
| Jenna | £250,000 | £4,626 | £254,626 |
| **Total** | **£666,300** | **£66,630** | **£732,930** |

Table 5

This is a slight improvement on the Equal Pay Rise.

Now Andrew gets a bigger pay rise than Jenna. This accelerates the closing of the pay gap.

Jenna gets the smallest raise in the whole company.

The ratio between employees pay will reduce slight faster over time. (i.e. Hayleigh earns 3 times more than Beatrice with their Current Salaries but only 2.45 times more under their New Salaries (vs 2.5 time under Equal Pay Rise)).

The pay gap between all employees will now close over time eventually (i.e. Frank used to earn £10,000 more than Emma but now only earns £9,888.89 more).

Again it is interesting to view these raises as percentages:

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Current Salary** | **Employee**  **Raise** | **Raise as %** |
| Andrew | £16,300 | £7,222 | 44.31% |
| Beatrice | £20,000 | £7,181 | 35.91% |
| Cason | £30,000 | £7,070 | 23.57% |
| Dalton | £30,000 | £7,070 | 23.57% |
| Emma | £30,000 | £7,070 | 23.57% |
| Frank | £40,000 | £6,959 | 17.40% |
| Gilbert | £40,000 | £6,959 | 17.40% |
| Hayleigh | £60,000 | £6,737 | 11.23% |
| Ian | £150,000 | £5,737 | 3.82% |
| Jenna | £250,000 | £4,626 | 1.85% |
| **Total** | **£666,300** | **£66,630** |  |

Table 6

We can see here that again Ian and Jenna have suffered a little more under this scheme but that every other employee has come off better.

By scraping just parts of a percent from Ian and Jenna’s raise the company has been able to give all other Employees a bigger increase in pay. This might seem unfair but refer to Table 2 and see that even Jenna is still getting a bigger absolute pay rise than Gilbert would have done in the original 10% for all scheme.

## Option 3. Variable Speed, Raise of Others

It is possible to increase the speed of the scheme to bring all salaries to an equilibrium.

Taking Equation 3 we can add a weighting value *k*:

Equation 5

Here k can be used to adjust how quickly the New Salaries will come into alignment.

The weight *k* basically removes the employees salary multiple times from the mean amplifying the sum of everyone else’s salary

For the case of *k = 0* then the equation simplifies as shown:

Equation 6

This is same results as for Equation 2 where all employees get the same absolute raise.

For the case of *k = 1* then the equation will match Equation 3.

As *k* is increased to the point of *k = N-1* the salaries converge faster.

Equation 7

At this point is takes only a few pay “rises” to get all employees to equal pay. But Jenna will not be happy as her pay rise will be negative for k values bigger than 3. This is due to her salary *S(Jenna)* making up more than a third of the salary total *STotal*.

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **New Salary**  **K = 2** | **New Salary**  **K = 4** | **New Salary**  **K = 6** |
| Andrew | £24,221 | £26,318 | £30,513 |
| Beatrice | £27,829 | £29,772 | £33,658 |
| Cason | £37,579 | £39,105 | £42,158 |
| Dalton | £37,579 | £39,105 | £42,158 |
| Emma | £37,579 | £39,105 | £42,158 |
| Frank | £47,329 | £48,438 | £50,658 |
| Gilbert | £47,329 | £48,438 | £50,658 |
| Hayleigh | £66,829 | £67,105 | £67,658 |
| Ian | £154,579 | £151,105 | £144,158 |
| Jenna | £252,079 | £244,438 | £229,158 |
| **Total** | **£732,930** | **£732,930** | **£732,930** |

Table 7

If giving negative pay rises seems unfair then we need to bound k with a different maximum:

Equation 8

So the maximum *k* value is equal to the ratio of the largest salary to the total salary up to when that ratio reaches *N-1*.

We could decide to set k to its maximum value each time:

Equation 9

*k* will reach *N-1* when all salaries are equal, only then will the previous top salary earner get a raise. Until that point their salary will be stable.

Below shows the year on year maximum k value (increasing each year) with the resulting salaries each year.

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Year 1 Salary**  **K = 2.67** | **Year 2 Salary**  **K = 2.93** | **Year 3 Salary**  **K = 3.22** |
| Andrew | £24,792 | £34,133 | £44,408 |
| Beatrice | £28,357 | £37,550 | £47,663 |
| Cason | £37,994 | £46,787 | £56,460 |
| Dalton | £37,994 | £46,787 | £56,460 |
| Emma | £37,994 | £46,787 | £56,460 |
| Frank | £47,631 | £56,024 | £65,257 |
| Gilbert | £47,631 | £56,024 | £65,257 |
| Hayleigh | £66,904 | £74,498 | £82,852 |
| Ian | £153,634 | £157,631 | £162,027 |
| Jenna | £250,000 | £250,000 | £250,000 |

Table 8

By not giving Jenna any pay rise for a few years it’s been possible to take Andrew from the original £16k to over £44k in four years. Under the normal percentage scheme his salary would only have increased to £21.7k in this time (where as Jenna would have expected to have increased her salary to £332k).

Under this scheme, with the given table of employees, it takes 15 pay rises of 10% for all salaries to match. For a constant *k* value of 1 this would take over 100 years, for *k* value of 0 the salary’s never converge.

## Option 3.1 Variable Speed, Raise of Others, as a Percentage of Max k

This option is just a dressed-up option 3 but replaces *k* with a *kp* (a percentage of max *k*) making it more usable.

Here *k* is tied to a percentage value *p*:

Equation 10

Each time salary rises are due *k* is recalculated and a fixed percentage is taken from it. The closer *p* is to 0 the more equally the absolute pay rise is, the closer *p* is to 1 the more skewed the raise is in favour or lower salary worker (i.e. the salaries will converge faster).

# Pay CUts

Sometimes Company X might have a bad few years and need to make some pay cuts in order to keep all its employees.

If we were to use Equation 10 with a negative pay rise *Rt* then we will hurt lowest paid employees worst. This is clearly not fair.

Here using a simple matched percentage cut to each employee may seem to be fairer. But there is an alternative.

First, we need to adjust k to be limited by the lowest employee salary:

Equation 11

Now when *p = 1* this results in the lowest paid employee not receiving any pay cut and the highest employees carrying most of the cut.

The following shows a 10% pay cut *(Rt = -10%)* using the above equation with *p = 1*, *k* is calculated as 40.88.

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Current Salary** | **Employee**  **Raise** | **Raise as %** |
| Andrew | £16,300 | £0 | 0.00% |
| Beatrice | £20,000 | -£490 | -2.45% |
| Cason | £30,000 | -£1,814 | -6.05% |
| Dalton | £30,000 | -£1,814 | -6.05% |
| Emma | £30,000 | -£1,814 | -6.05% |
| Frank | £40,000 | -£3,138 | -7.84% |
| Gilbert | £40,000 | -£3,138 | -7.84% |
| Hayleigh | £60,000 | -£5,785 | -9.64% |
| Ian | £150,000 | -£17,700 | -11.80% |
| Jenna | £250,000 | -£30,939 | -12.38% |
| **Total** | **£666,300** | **-£66,630** |  |

Table 9

The control variable *p* is now bound differently:

Equation 12

As *p* tends to infinity the pay cut percentage “Raise as %” in the table above become more equal.

Equation

For example, if *p = 5* then the resultant raises are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| **Name** | **Current Salary** | **Employee**  **Raise** | **Raise as %** |
| Andrew | £16,300 | -£1,371 | -8.41% |
| Beatrice | £20,000 | -£1,760 | -8.80% |
| Cason | £30,000 | -£2,812 | -9.37% |
| Dalton | £30,000 | -£2,812 | -9.37% |
| Emma | £30,000 | -£2,812 | -9.37% |
| Frank | £40,000 | -£3,863 | -9.66% |
| Gilbert | £40,000 | -£3,863 | -9.66% |
| Hayleigh | £60,000 | -£5,966 | -9.94% |
| Ian | £150,000 | -£15,429 | -10.29% |
| Jenna | £250,000 | -£25,943 | -10.38% |
| **Total** | **£666,300** | **-£66,630** |  |

Table 10

# Summary

At this point the findings can be summarised as:

Equation

Selecting values for *p* in each case is an interesting business decision about fairness. It is suggested that *p* could be derived to either:

1. Be a fixed value.
2. Ensure the range of absolute raises is limited.
3. Limit the maximum raise percentage of the lowest earner as a multiple of the company wide raise.

# Further work

1. Find a way to measure “fairness” as numerical metrics
2. Find a method to derive p based on “fairness” as defined above
3. Consider “pay per performance” impact on this system.
4. Consider how this works with regards to existing pay structures (often referred to as “bands”)

1. For the purposes of holding the readers interest, and amusing the author, we with discuss this problem from the perspective of a company that sells X’s for Noughts & Crosses boards. They have chosen to call themselves “Company X”. [↑](#footnote-ref-1)
2. Company X has a strange policy of paying people based strongly on their first names. [↑](#footnote-ref-2)
3. Andrew has a daughter that wants to go to college to learn how to make pieces for a new game called Chess. [↑](#footnote-ref-3)